Machine Learning for NLP

Supervised Learning

Aurélie Herbelot

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Centre for Mind/Brain Sciences
University of Trento

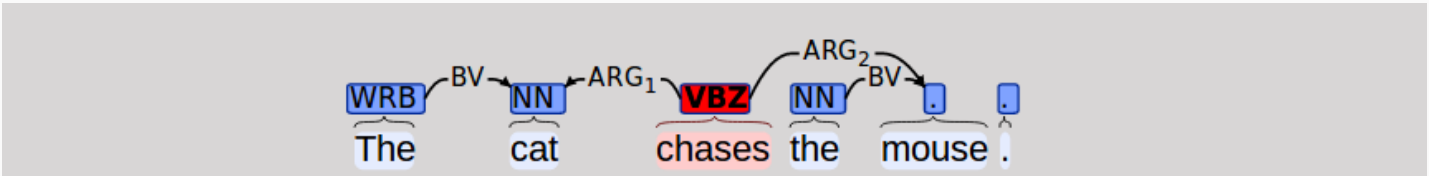
Linear Regression

Difference between regression and classification

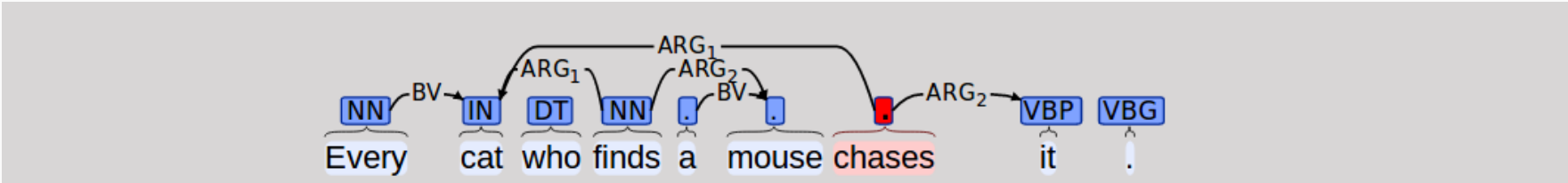
- Naive Bayes is a *classification* algorithm: given an input, we want to predict a discrete class, e.g.:
  - Austen vs Carroll vs Shakespeare;
  - bad vs good (movie review);
  - spam vs not spam (email)...
- In *regression*, given an input, we want to predict a continuous value.

Linear regression example

- Let's imagine that reading speed is a function of the structural complexity of a sentence.



58 edges



225 edges

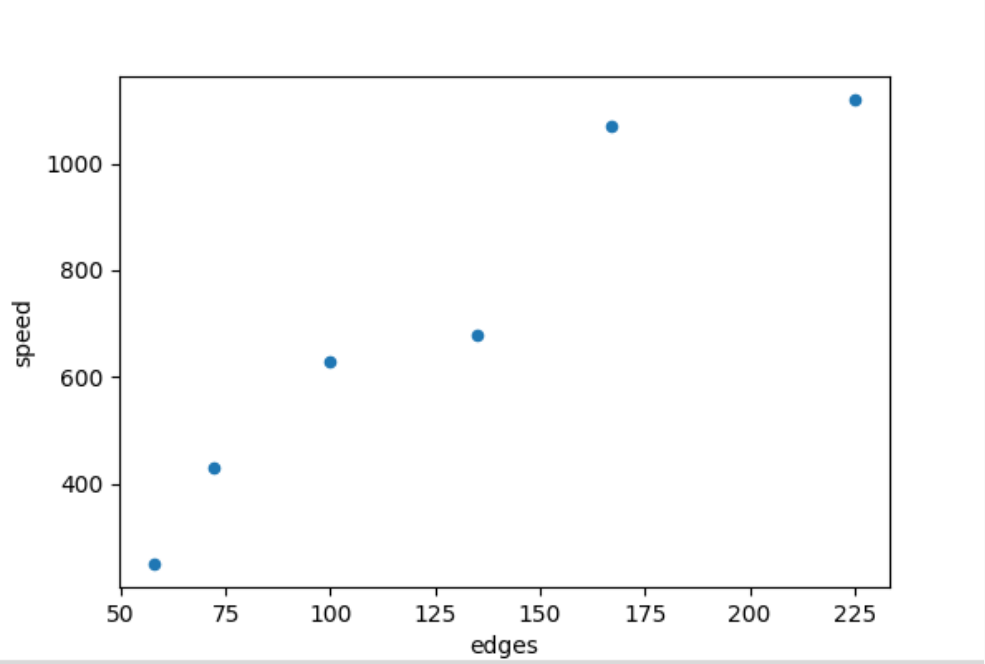
Parses from `http://erg.delph-in.net/logon`.

Linear regression example

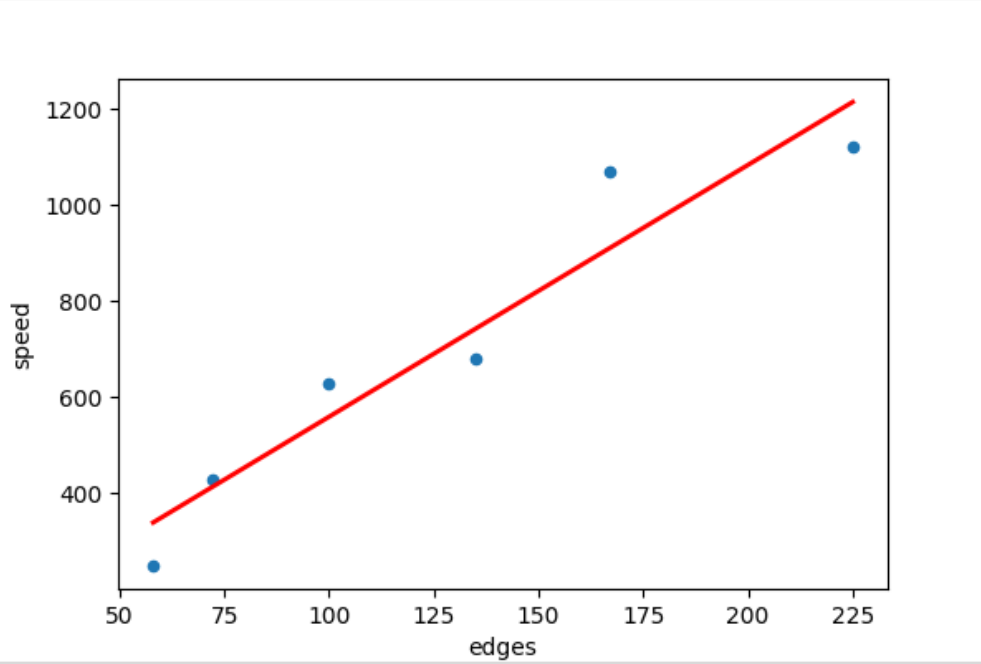
	#Edges	Speed (ms)
Sentence 1	58	250
Sentence 2	100	720
Sentence 3	72	430
Sentence 4	135	1120
Sentence 5	225	1290
Sentence 6	167	1270

- Let's call the edges feature x and the speed output y.
- We want to predict the *continuous* value y from x.
- Example: if a new sentence has 240 edges, can we predict the associated reading speed?

Linear regression example

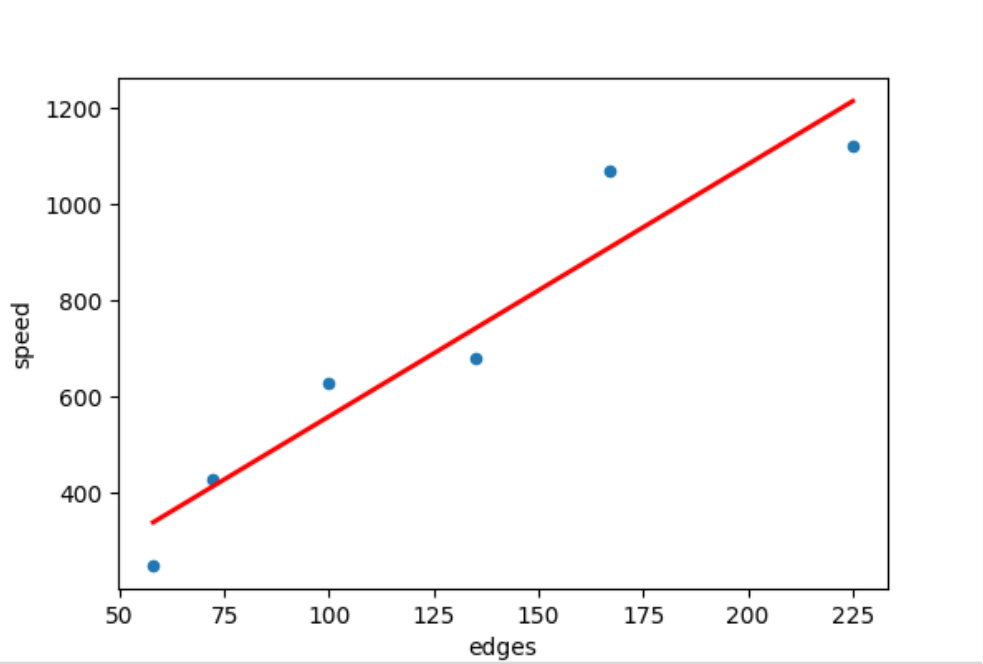


Linear regression example



We want to find a linear function that models the relationship between x and y.

Linear regression example

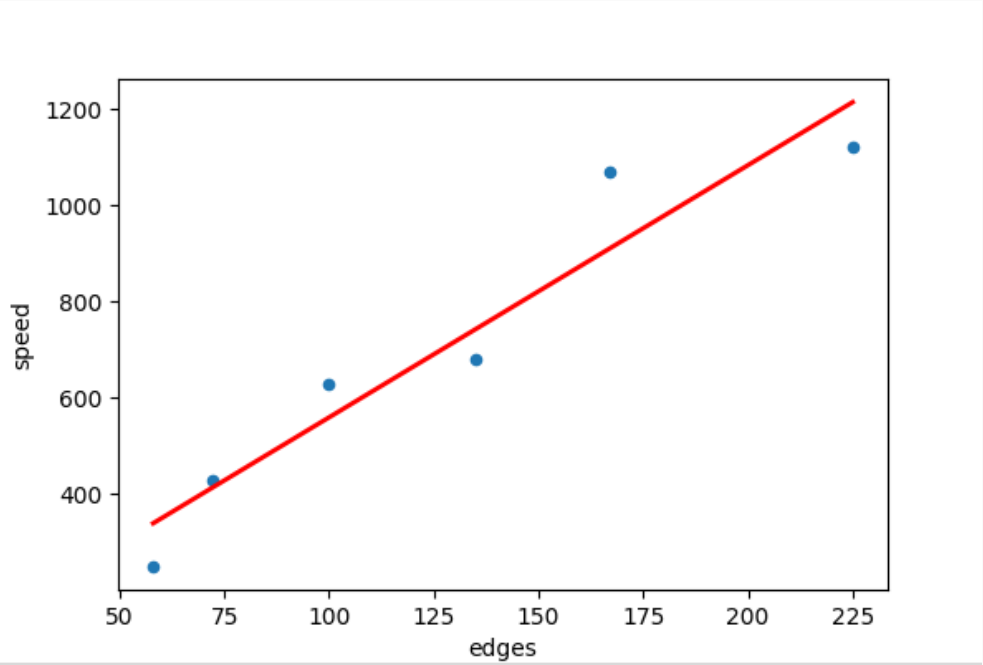


This linear function will have the following shape:

$$y = \theta_0 + \theta_1 x$$

θ_0 is the *intercept*, θ_1 is the *slope* of the line.

Linear regression example



Let's say our line can be described as $y = 36 + 5x$.

Now we can predict a reading speed for 240 edges:

$$speed = 36 + 5 \times 240 = 1236ms.$$

How do we find the line?

- We estimate θ_0 and θ_1 using the *least squares* criterion.
- This means our ideal line through the data will minimise the *sum of squared errors*:

$$E = \frac{1}{2N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

where N is our number of training datapoints, $\hat{y}_i$ is the model prediction for datapoint i, and y_i is the gold standard for i.

The Gradient Descent Algorithm

Minimising the error function

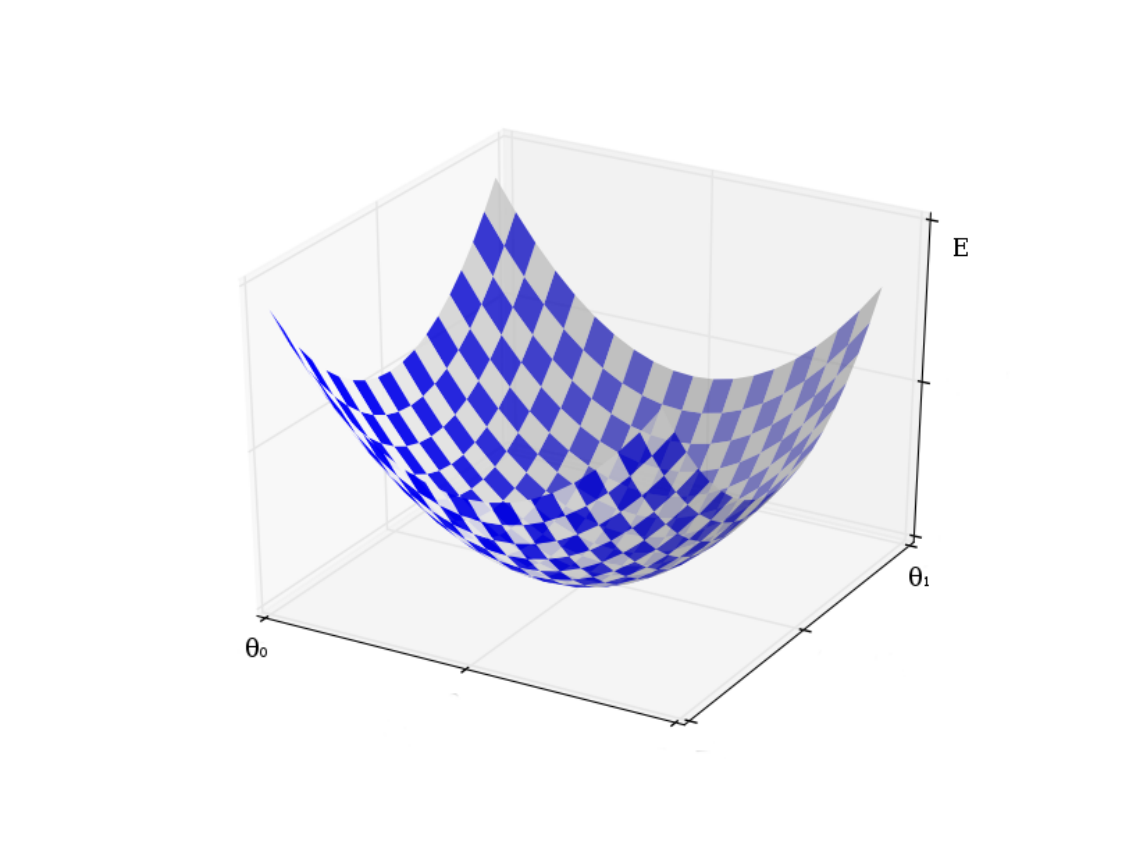
$$E = \frac{1}{2N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2 = \frac{1}{2N}\sum_{i=1}^{N}(\theta_0 + \theta_1 x_i - y_i)^2$$

E is a function of θ_0 and θ_1.

It is calculated over *all* training examples in our data (see $\sum$).

How do we find its minimum *min* $E(\theta_0, \theta_1)$?

Gradient descent



In order to find *min* $E(\theta_0, \theta_1)$, we will randomly initialise our θ_0 and θ_1 and then 'move' them in what we think is the right direction to find the bottom of the plot.

What is the right direction?

To take each step towards our minimum, we are going to update θ_0 and θ_1 according to the following equation:

$$\theta_j := \theta_j - \alpha \frac{\delta}{\delta \theta_j} E(\theta_0, \theta_1)$$

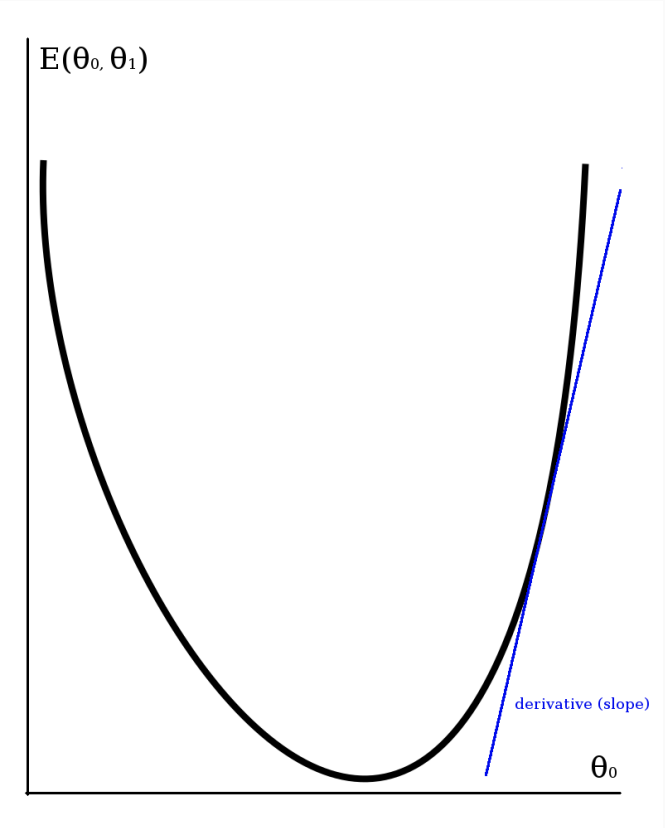
α is called the *learning rate*.

$\frac{\delta}{\delta \theta_j} E(\theta_0, \theta_1)$ is the *derivative* of E for a particular value of θ.

(j in the equation simply refers to either 0 or 1, depending on which θ we are updating.)

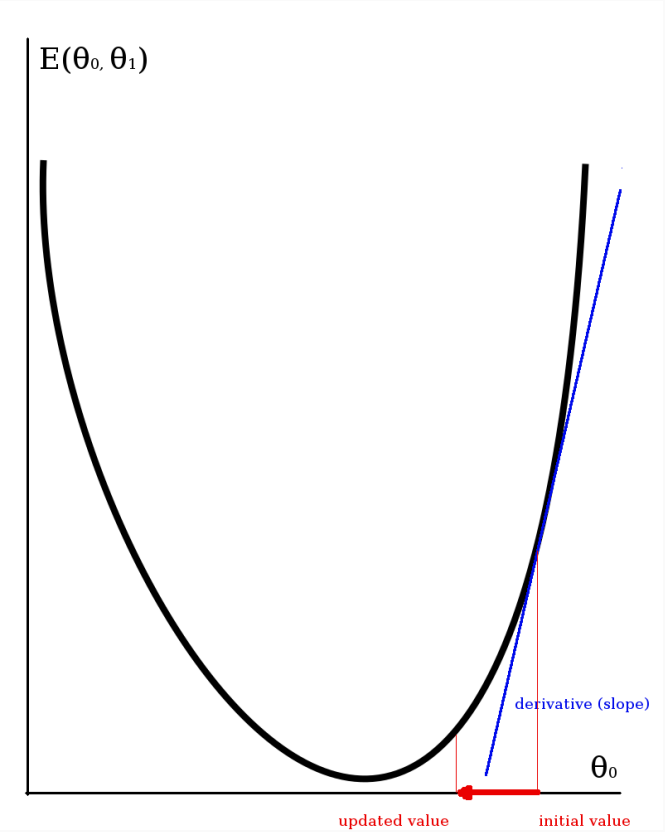
What does the derivative do?

- Imagine plotting just one θ, e.g. θ_0, against the error function.
- We have initialised θ_0 to some value on the horizontal axis.
- We now want to know whether to increase or decrease its value to make the error smaller.



What does the derivative do?

- The derivative of E at θ_0 tells us how steep the function curve is at this point, and whether it goes 'up or down'.
- Effect of positive derivative D_+ on the θ_0 update:
  $\theta_0 := \theta_0 - \alpha D_+$
  θ_0 decreases!

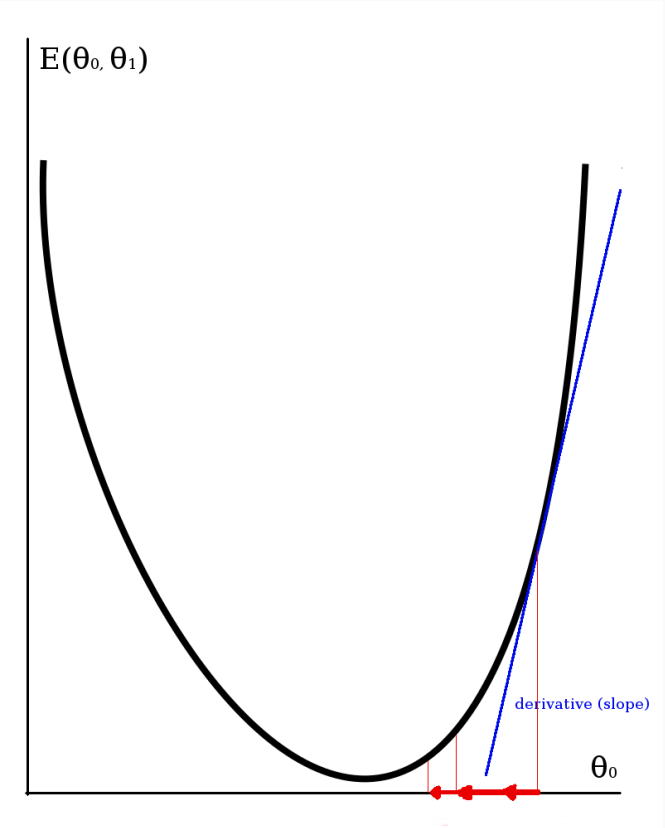


What does the learning rate do?

- α multiplies the value of the derivative, so the bigger it is, the bigger the update to θ:

$$\theta_j := \theta_j - \alpha \frac{\delta}{\delta \theta_j} E(\theta_0, \theta_1)$$

- A too small α will result in slow learning.



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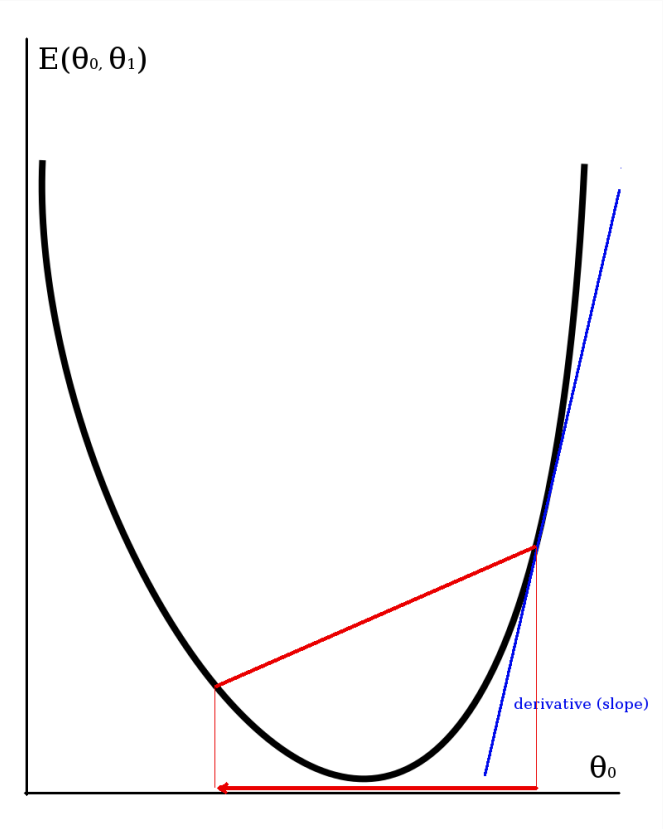
$$\theta_j := \theta_j - \alpha \frac{\delta}{\delta \theta_j} E(\theta_0, \theta_1)$$

- A too large α may result in **not** learning.



Putting it all together

- The gradient descent algorithm finds the parameters θ of the linear function so that prediction errors are minimised with respect to the training instances.
- We do repeated updates of both θ_0 and θ_1 over our training data, until we *converge* (i.e. the error does not go down anymore).
- The final θ values after seeing all the training data should be the best possible ones.

To bear in mind...

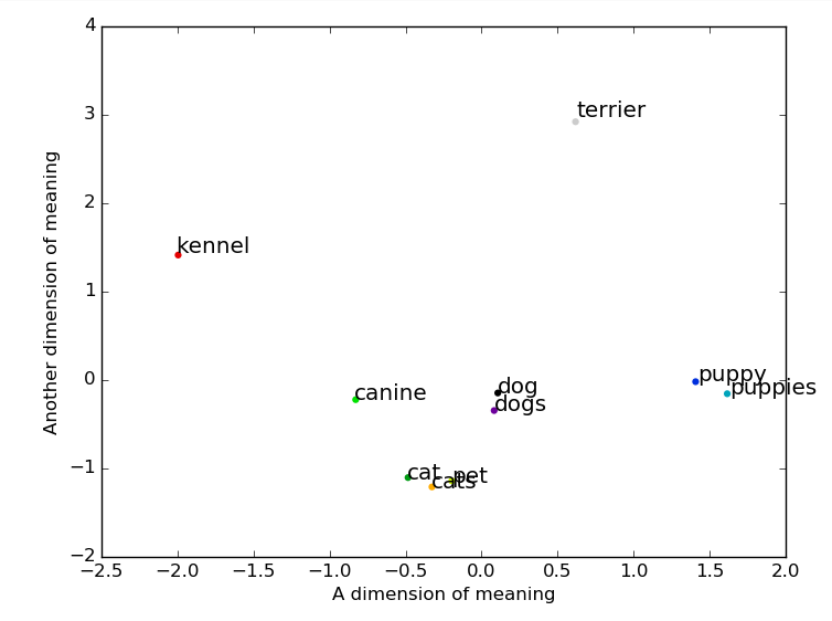
- How well and how fast gradient descent will train depends on how you initialise your parameters.
- Can you see why? (Hint: come back to the error curve and imagine a different starting value for θ_0.)

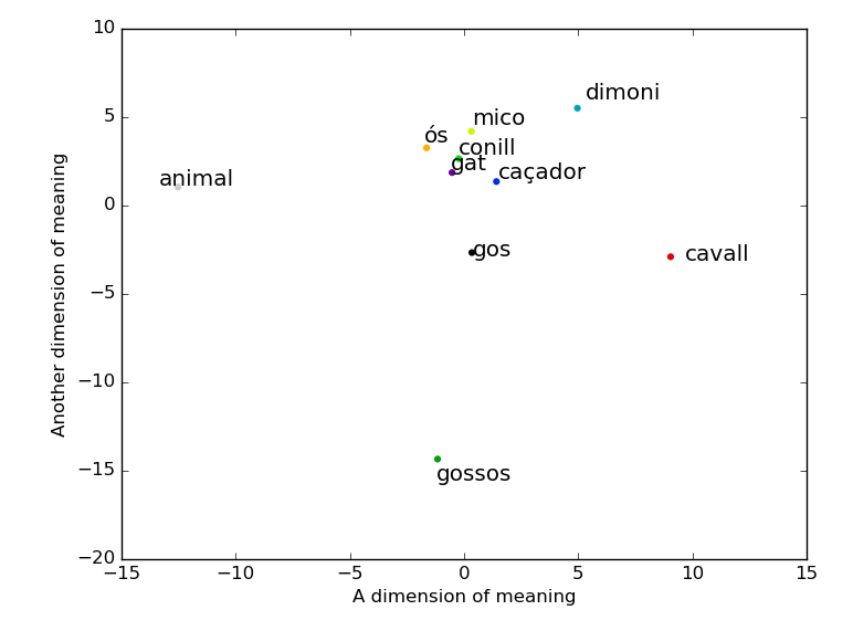
Partial Least Square Regression

Regression as mapping

- We can think of linear regression as *mapping* from one dimension to another: from the values on the x-axis to the values on the y-axis.
- Partial Least Square Regression (PLSR) allows us to define such a mapping from matrices to matrices, via dimensionality-reduced spaces.

Example matrix-to-matrix mapping

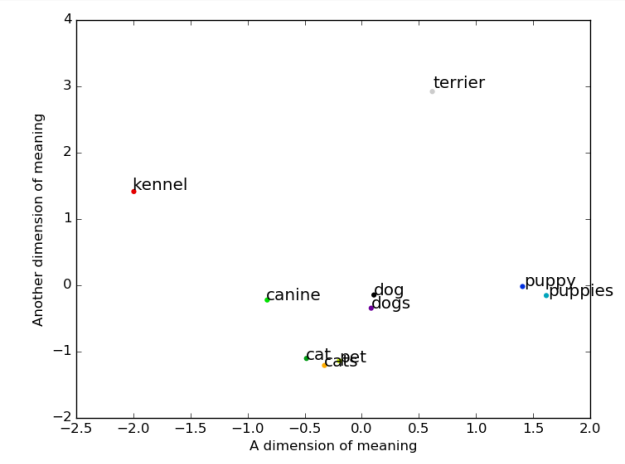


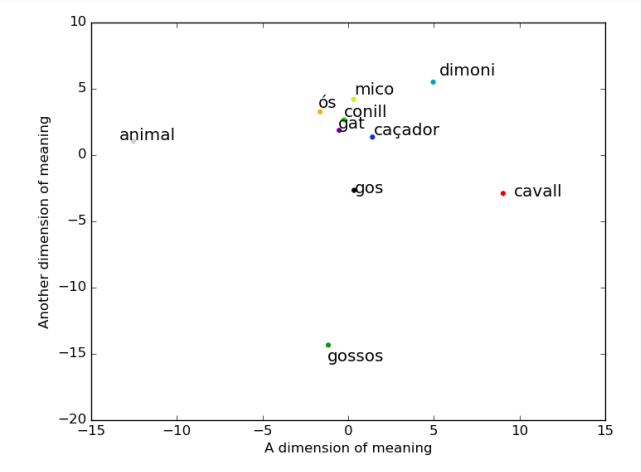


Can we map an English semantic space into a Catalan semantic space?

Example matrix-to-matrix mapping

- Here, each datapoint in both input and output is represented in hundreds of dimensions.
- The dimensions in space 1 are not the dimensions in space 2.
- Intuitively, translation involves a recourse to meta-linguistic concepts (some *interlingua*), but we don't know what those are.

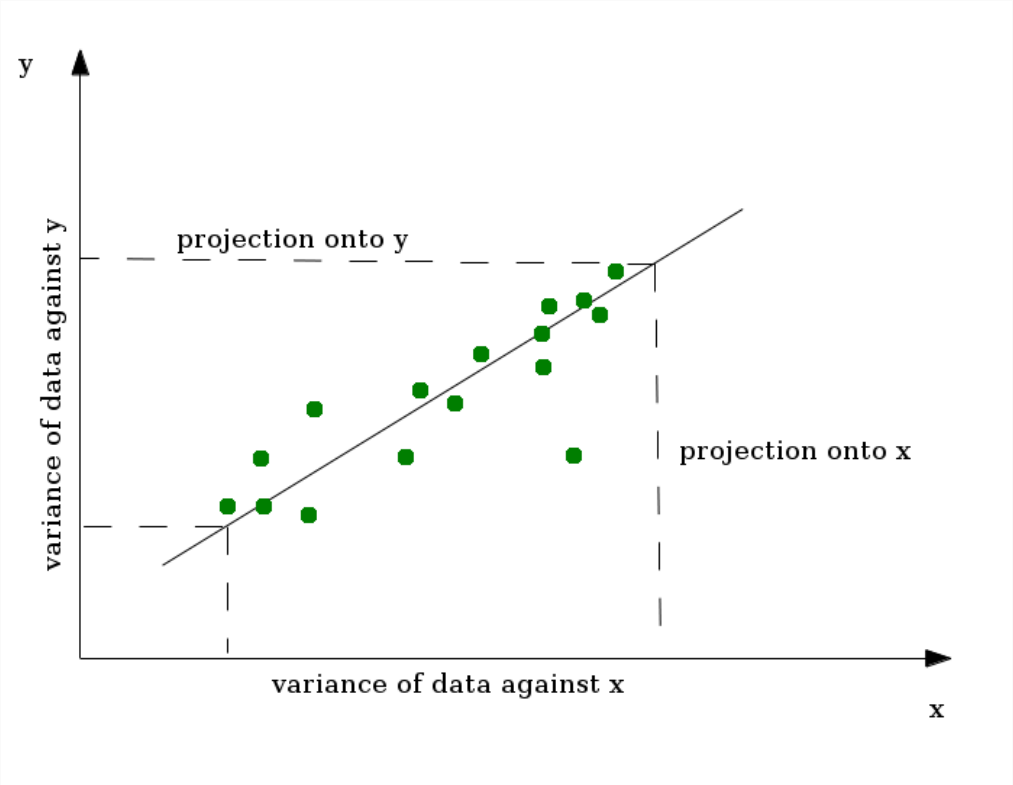




Principal Component Analysis

- Let's pause for a second and look at the notion of Principal Component Analysis (PCA).
- PCA refers to the general notion of finding the components of the data that maximise its variance.
- Let's now look at a graphical explanation of variance. It will be useful for our understanding of PLSR.

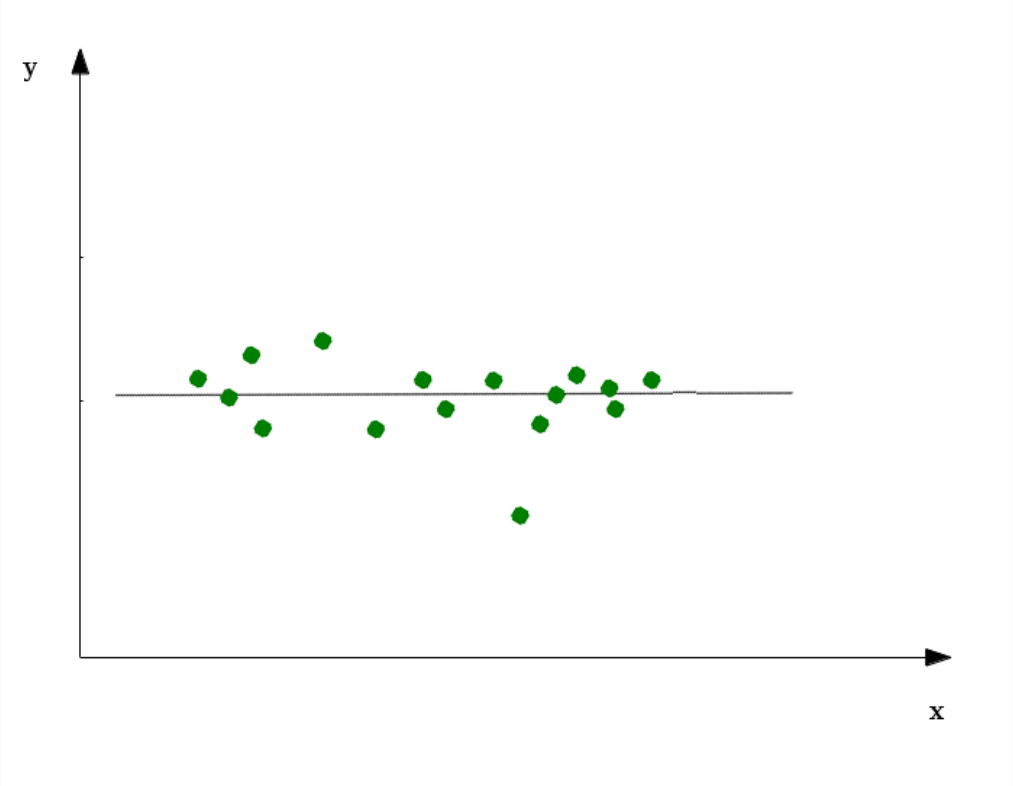
(Non-)explanatory dimensions



If we project these green datapoints on the *x* axis, we still explain a lot about the distribution of the data. A little less so with the *y* axis.

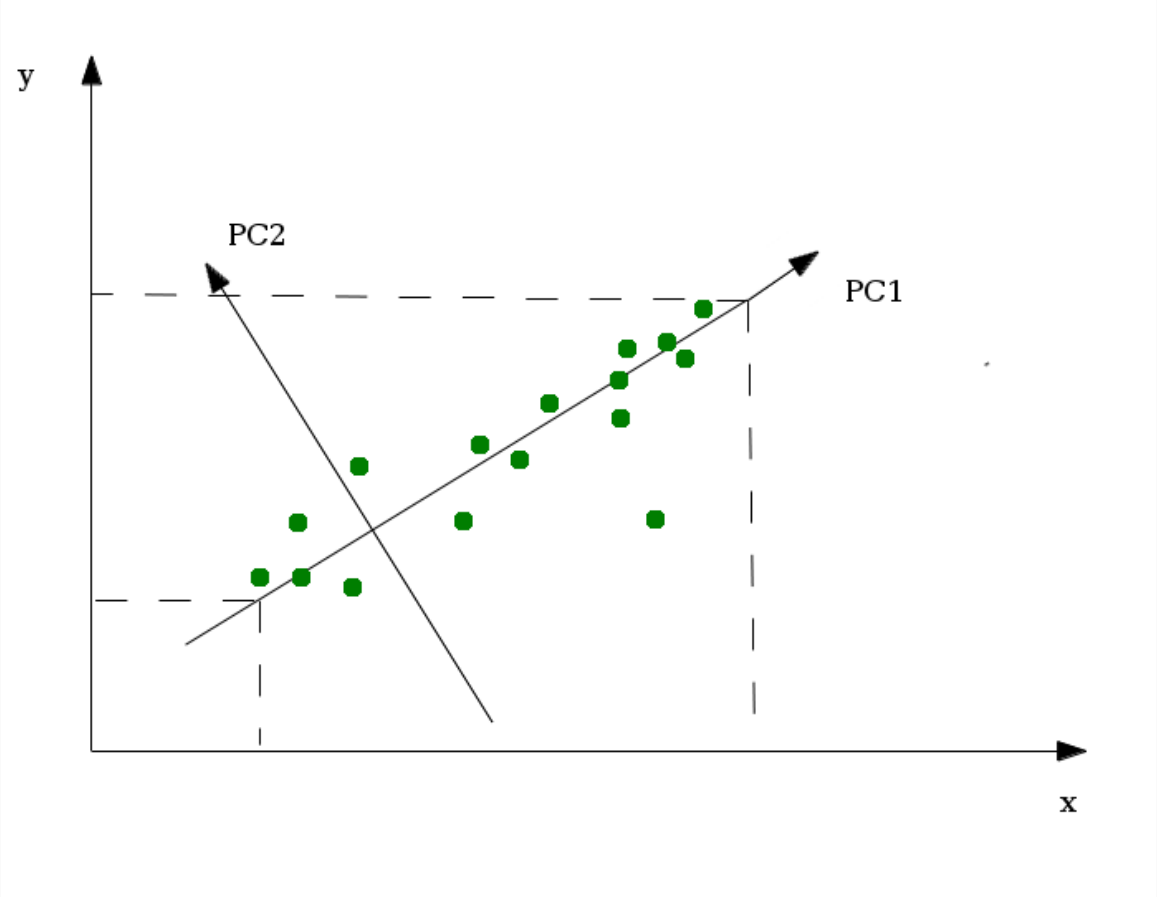
x* explains more of the variance than *y

(Non-)explanatory dimensions



Here, the *y* axis is rather uninformative. Get rid of it?

(Non-)explanatory dimensions



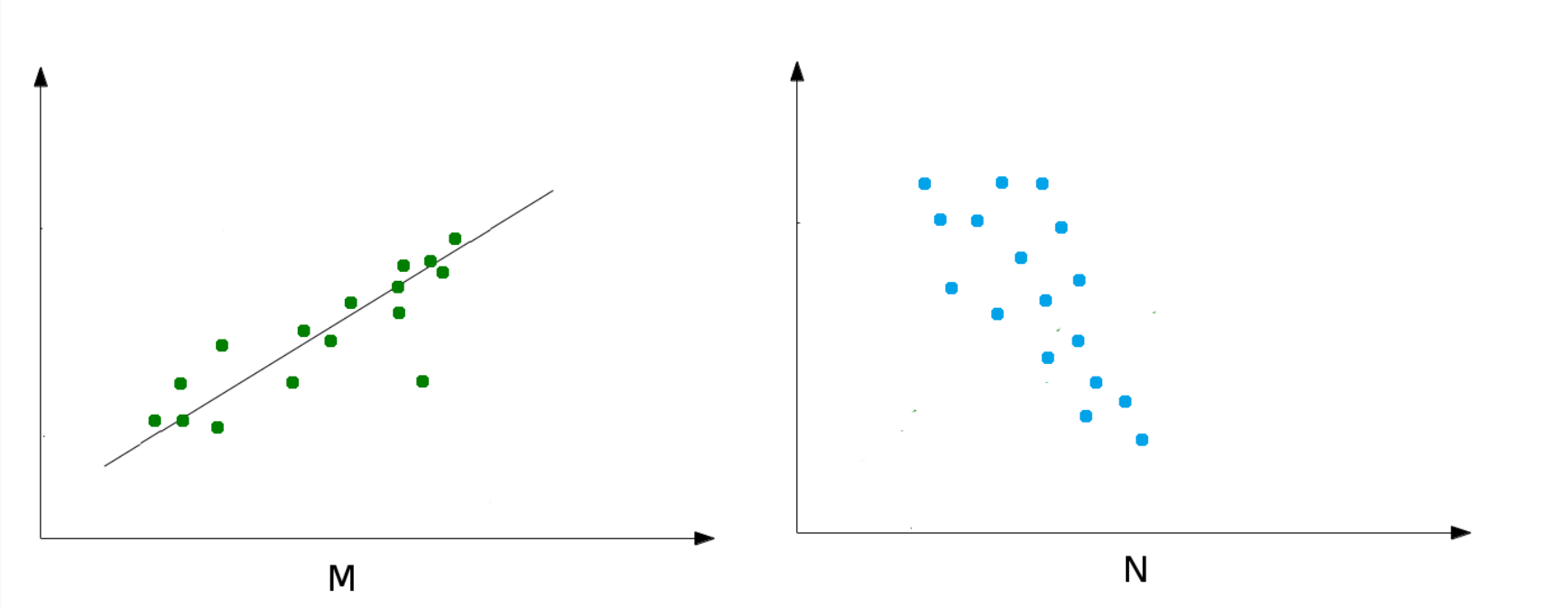
Actually, here are the most informative dimensions...
we'll call them PC1 and PC2.

Back to PLSR...

- We use PLSR when we want to predict a set of features from another set of features.
- In effect, we want to predict a matrix N of size k x q from a matrix M of size k x p.
- For instance, we might have:
  - a matrix M with $k = 3$ English words (*dog, cat, eat*), expressed in $p = 300$ dimensions;
  - a matrix N with $k = 3$ Catalan words (*gos, gat, menjar*), expressed in $q = 400$ dimensions.

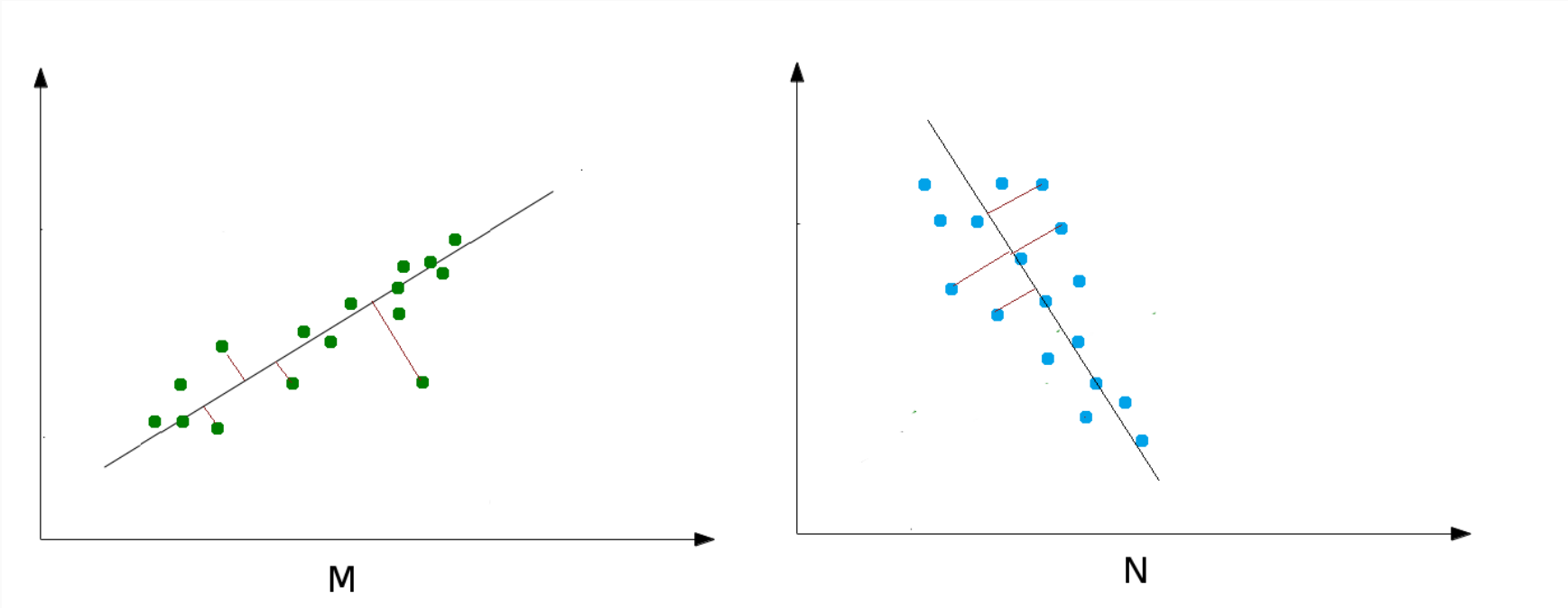
PLSR: intuition

- Let's assume we have two sets of data (the matrices *M* and *N*) and we perform PCA on both of them to find their principal components.
- Warning: it is not exactly what happens in PLSR, but it will make things easier to understand...



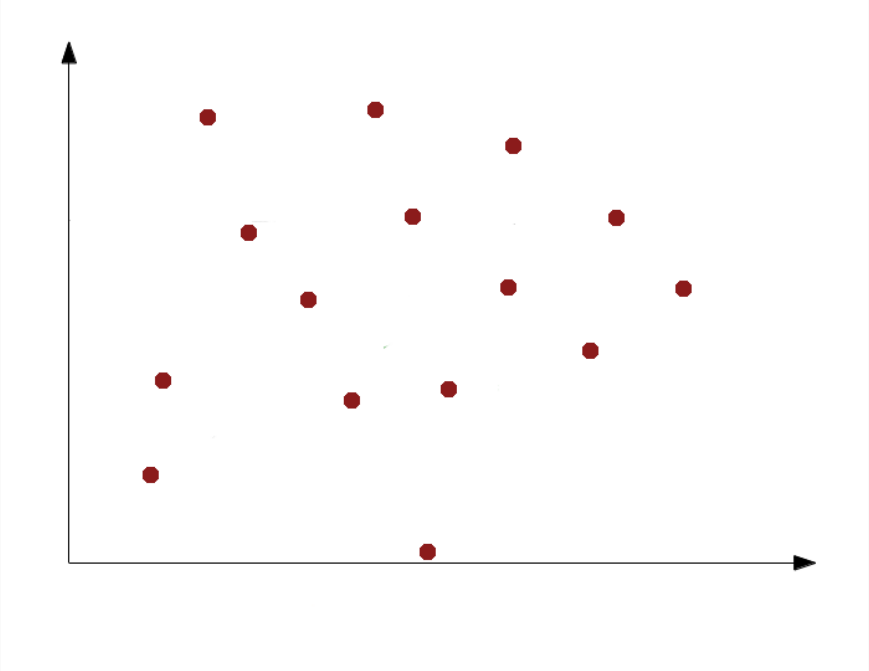
PLSR: intuition

- Now we have PC1 for each matrix, we can try to find a linear function that maps the data in PC1 of *M* into the data in PC1 of *N* (linear regression problem).
- We first need to replot our data onto PC1, using projection.



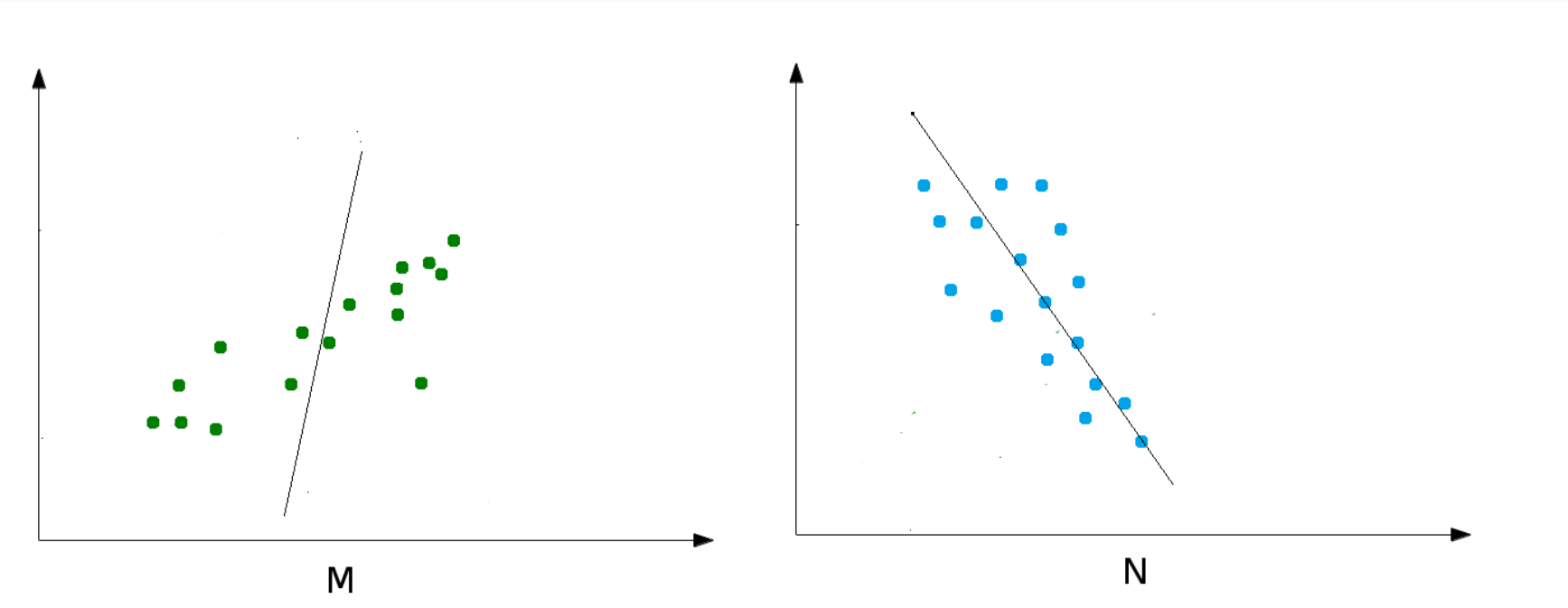
PLSR: intuition

- Once data has been replotted onto a component for both matrices, we can plot the result onto a 2D graph.
- On the resulting graph, the x dimension will receive the values of the points in *M* projected onto PC1 of *M*. The y dimension receives the values of the points in *N* projected onto PC1 of *N*.
- Problem: that plot might not show very much correlation...



PLSR: what actually happens

- For a given component, we want to find the function that maximises the covariance (correlation) between the two matrices.
- To do this, we are going to keep PC1 of *N* and rotate PC1 of *M* until we find that maximum covariance.



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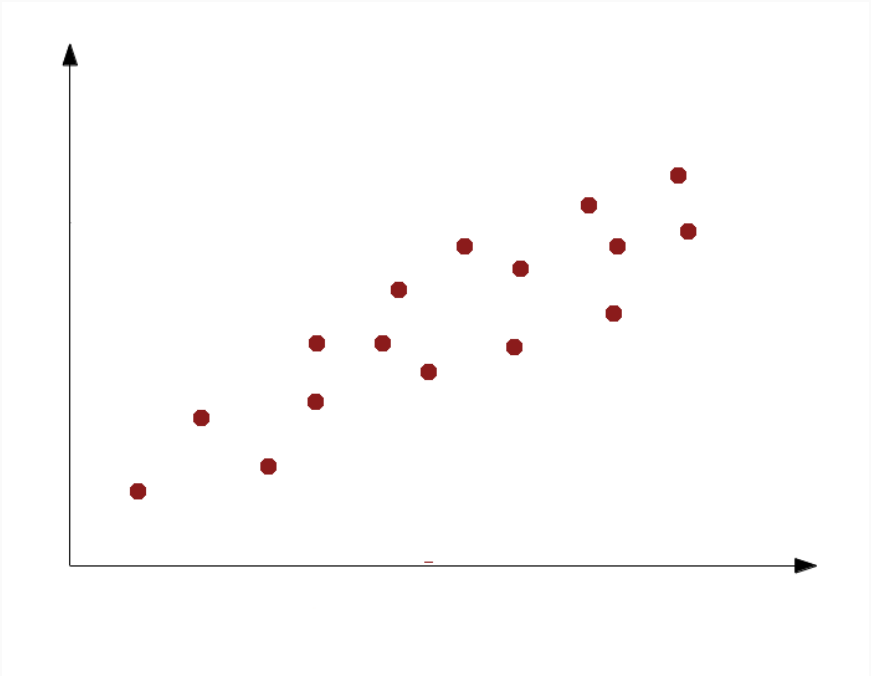


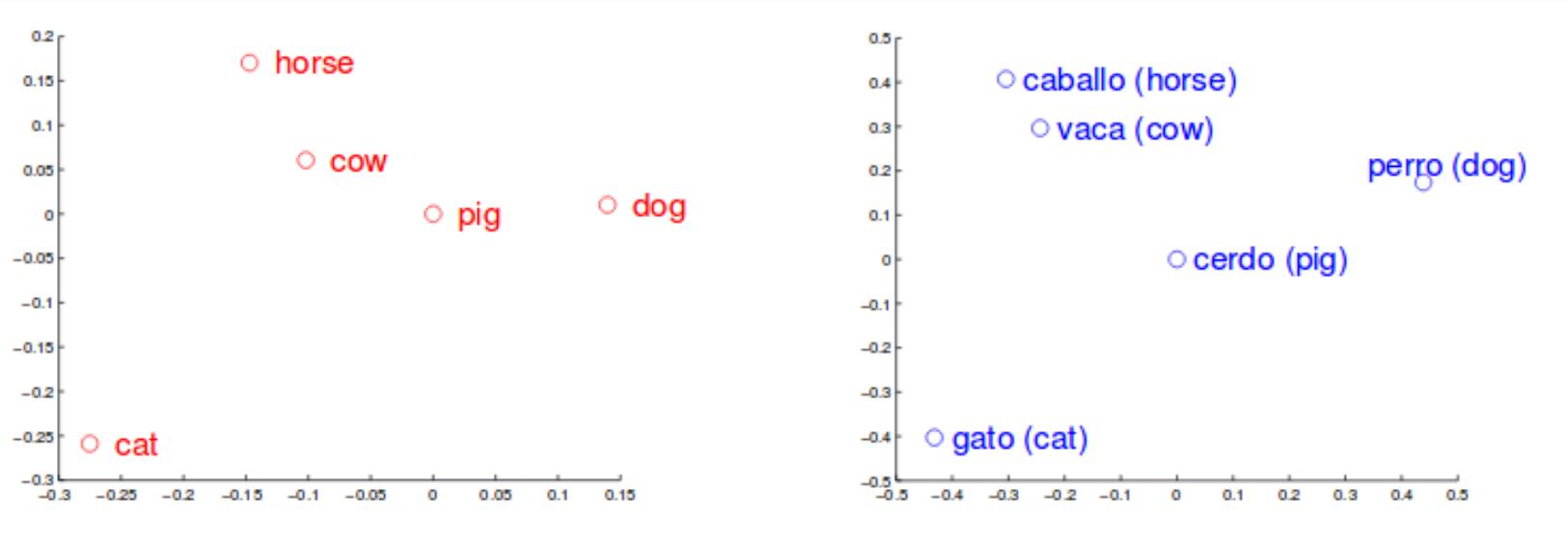
PLSR in real life

- In effect, running PLSR on two matrices M and N will give us a learnt projection matrix A which, when multiplied by M gives us and approximation of N.
- We have looked at what happens with just one component, but in practice, we'll choose a higher number of components.
- Is there a way to interpret the resulting, dimensionality-reduced representations of both original matrices? (More on this in the unsupervised learning session...)

Translating with regression

- Let's say we have two semantic spaces S_1 and S_2.
- We assume we also have a set of overlapping *seeds* $O = \{w_1, w_2 ... w_n\}$, for which vectors are available in both spaces.
- We can extract two matrices from S_1 and S_2 corresponding to the words in O:
  $O_1 \subset S_1$ and $O_2 \subset S_2$.
- Learn the regression.
- Apply to $S_1 - O_1$, the words for which we don't have a mapping.

Translating single words



Mikolov et al (2013). https://arxiv.org/pdf/1309.4168.pdf

Distance as confidence measure

- There may be some general regularity in translation, but some words' meanings will be idiosyncratic.
- How can we know whether a translation is good or not?
- Use distance to nearest neighbour: intuitively, if the translated vector y is quite far from every other word in the space's vocabulary V, it may not be a quality vector. So we impose the constraint:

$$max_{i \in V} cos(y, z_i) > t$$

where t is a threshold.

Distance as confidence measure

- Evaluation: for a predicted vector, see whether its nearest neighbour in the gold data is the correct translation.
- Effect of threshold vs precision @ N for English –> Spanish.
- Huge effect on recall.

Threshold	Coverage	P@1	P@5
0.0	92.5%	53%	75%
0.5	78.4%	59%	82%
0.6	54.0%	71%	90%
0.7	17.0%	78%	91%

Problems with aligning concepts

- Concepts are not universal: different languages/cultures associate different things with a word.
- The mapping is actually highly non-linear!

money	Distance
money	0
cash	0.36
earn	0.44
much	0.46
pay	0.47
buy	0.47
you	0.47
me	0.48
bank	0.48
winnings	0.49

geld	Distance	
geld	0	money
blut	0.35	broke
dollar	0.36	dollar
contanten	0.37	cash
platzak	0.38	broke
contant	0.38	in cash
verdienen	0.4	earn
betaald	0.41	paid
aanbetaling	0.41	deposit
betalen	0.42	pay

http://meshugga.ugent.be/snaut-dutch/

PLSR for semantic composition

- Baroni and Zamparelli (2010): Functional model for adjective-noun composition.
- Composition is the multiplication of vectors/matrices **learned** from access to phrasal distributions.
- 'Internal' evaluation: composition is evaluated against phrasal distributions.

Assumptions

- Given enough data, distributions for phrases should be obtained in the same way as for single words.
- I.e. it is fair to assume that if we have seen enough instances of *black cat*, the context of the phrase should give us an indication of its meaning (perhaps it is more related to witches than *cat* and *ginger cat*).
- Let's say we have a vector $\vec{a}$ (*black*) and a $\vec{n}$ (*cat*), and also a $\vec{an}$ (*black cat*), we can hypothesise a composition method which combines $\vec{a}$ and $\vec{n}$ to get $\vec{an}$ (standard machine learning).

Assumptions

- There is no single composition operation for adjectives. Each adjective acts on nouns in a different way:
  - *red car*: the outside of the car is evenly painted with the colour red (visual);
  - *fast car*: the engine of the car is powerful (functional);
  - *expensive car*: the price of the car is high (abstract/relational).
- Even single adjectives will combine with various nouns in different ways:
  - *red car*: outside of the car, even paint;
  - *red watermelon*: inside of the watermelon, probably not as red as the car;
  - *red nose*: a little redder than usual, probably due to a cold.

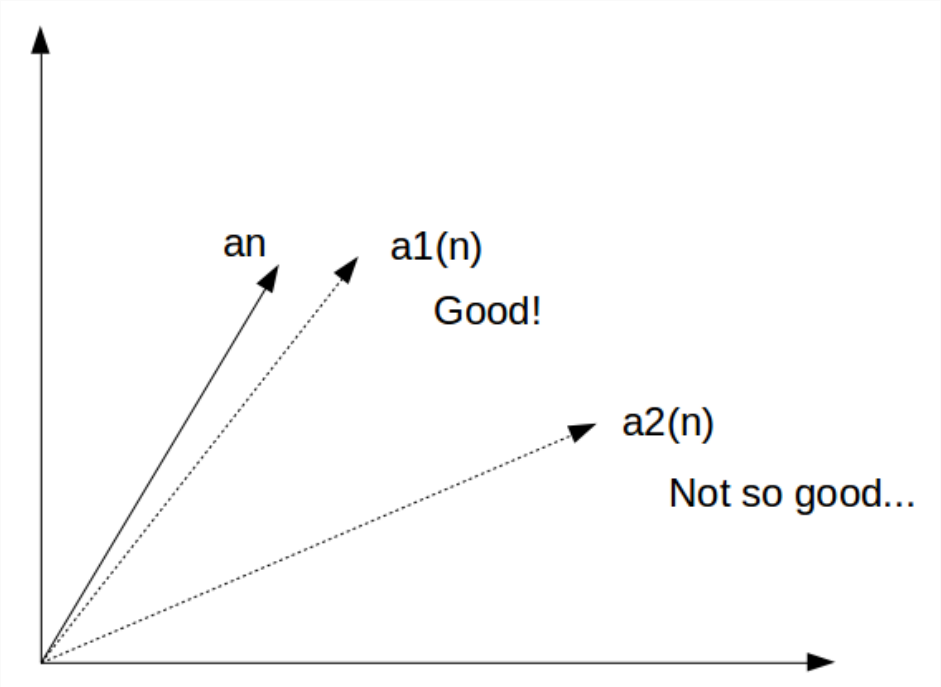
System

- In formal semantics, adjectives are seen as functions which ‘apply’ to nouns. They take a property (a noun phrase) and return another property (another noun phrase): $A(N) = AN$.
- For each adjective, a matrix is learned from actual AN phrases using PLSR.

$$\mathbf{AN} = \begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} an_1 + bn_2 + cn_3 \\ pn_1 + qn_2 + rn_3 \\ un_1 + vn_2 + wn_3 \end{pmatrix} = \begin{pmatrix} an_1 \\ an_2 \\ an_3 \end{pmatrix}$$

System

- Test by measuring distance between a given adjective-noun combination and the corresponding phrasal distribution on unseen data.



Learning the adjective matrix

- Assume you have a *gold standard*: data points for which you already have the solution to the task.
- In our AN setting, the gold standard is a number of adjective-noun phrases for which we have a) a noun vector; b) an adjective vector; c) a phrase vector (e.g. $\overrightarrow{black}$, $\overrightarrow{cat}$, $\overrightarrow{blackcat}$).
- Infer a rule (in our case, a matrix) which explains the observed data, using PLSR.
- Check whether the rule holds for unobserved data.

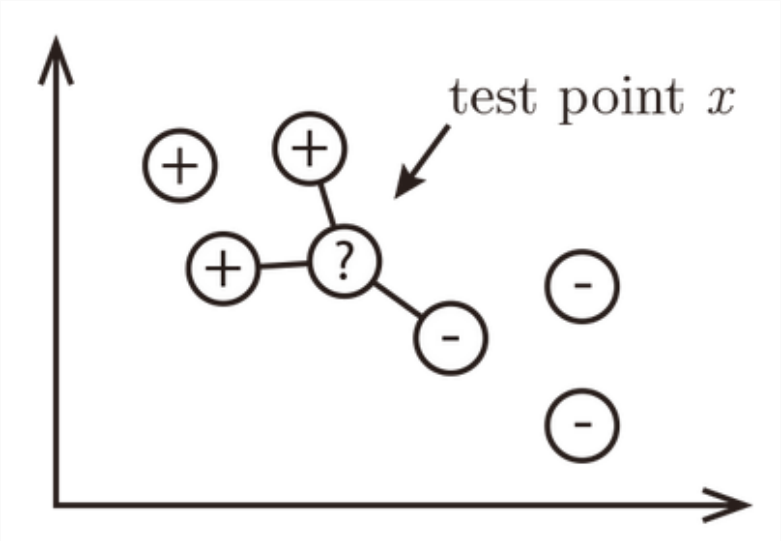
k-NN algorithm

The k-NN algorithm

- A simple but powerful algorithm using the position of training points in space.
- Let's assume some training data $\{(x_1, y_1), (x_2, y_2)...(x_n, y_n)\}$ where x_i is the input and y_i the output.
- y_i can take two forms:
  - $y_i \in \{1, ..., C\}$ in classification problems;
  - $y_i \in \mathbb{R}$ in regression problems (i.e y_i is a real value).
- We want to predict y_U from x_U, a new and unknown input.

The k-NN algorithm

The idea behind k-NN is that for a given x_i, we use its nearest neighbours for prediction.



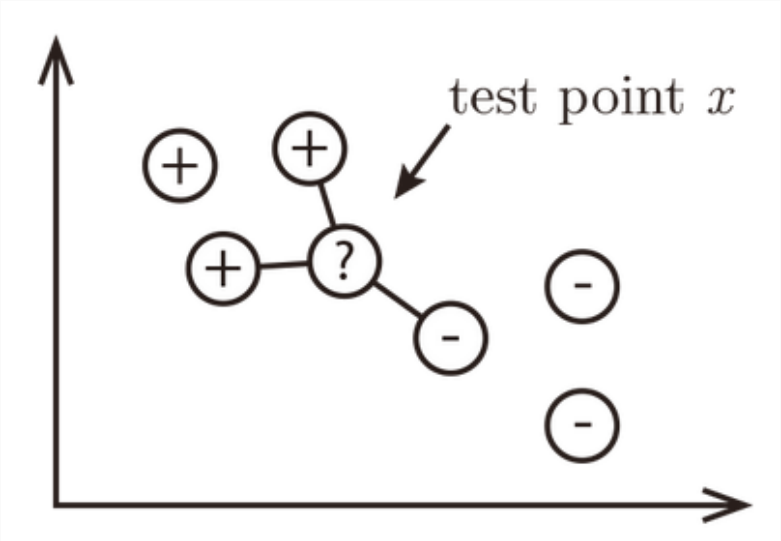
https://www.cs.cornell.edu/courses/cs4780/2017sp/lectures/lecturenote02_kNN.html

The k-NN algorithm

- The algorithm requires to set k, the number of neighbours, and a distance function d between points in the space.
- Four simple steps:
  - calculate $d(x_U, x_i)$ for all x_i in the data;
  - sort distances in descending order;
  - return top k distances;
  - y_U is given by *majority voting* for classification and *averaging* for regression.

Majority voting (classification)

Here, with $k = 3$ and two discrete classes, $+$ and $-$, we will return $y_U = +$ (majority class for the 3 nearest neighbours).



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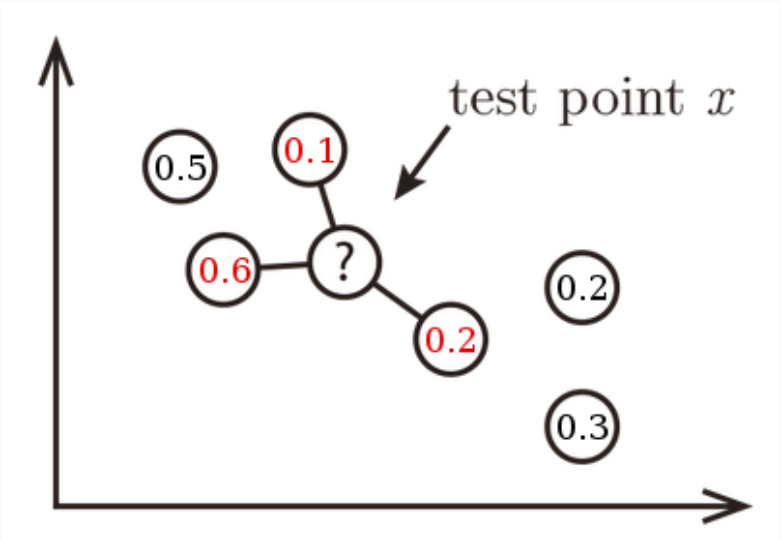
Averaging (regression)

Here, we simply average the y values of the nearest neighbours (shown inside each point). So $y_U = \frac{0.6+0.1+0.2}{3} = 0.3$.



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The distance function

- Traditionally, the Euclidian distance is used:

$$d(\mathbf{A}, \mathbf{B}) = \sqrt{\sum_{i=1}^{n} (A_i - B_i)^2}$$

- In distributional semantics, cosine is the measure of choice:

$$\cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2} \sqrt{\sum_{i=1}^{n} B_i^2}}$$

k-NN for word sense disambiguation

- Erk & Padó (2010): *Exemplar-Based Models for Word Meaning In Context*.
- In vector spaces, word representations are built out of *all* contexts in which the word occurs.
- So we will be conflating e.g. two senses of *coach:*
  - The *coach* drove a steady 45 mph.
  - The team has lost all games since the new *coach* arrived.
- Can we build sense-disambiguated word representations by only looking at the relevant exemplars?

Exemplars in cognitive science

- We can differentiate *prototype* vs *exemplar* models:
  - prototype models predict degree of category membership through similarity to a single prototype;
  - exemplar theory represents a concept as a collection of all previously seen exemplars.

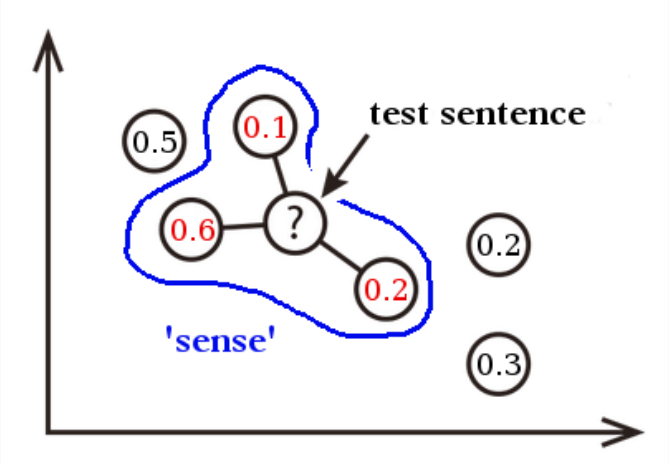
The k-NN disambiguation model

- We have a sentence s containing a word w to be disambiguated.
- We want to build a vector for w from the contexts most similar to s. Let's call these contexts the activation set E:

$$act(E, s) = \{e \in E | sim(e, s) > \theta(E, s)\}$$

- In k-NN activation, the k most similar exemplars to s are activated by setting θ to the similarity of the k-th most similar exemplar.

The k-NN disambiguation model



Given a sentence *s* expressed as a point in space and containing target word *T*, find the *k* closest sentences (also containing *T*). The union of those *k* sentences represents the *sense* of *T* in *s*.

Evaluation of the k-NN disambiguation model

Sentential context	Paraphrase
After a fire extinguisher is used, it must always be **returned** for recharging and its use recorded.	bring back (3), take back (2), send back (1), give back (1)
We **return** to the young woman who is reading the Wrigley's wrapping paper.	come back (3), revert (1), revisit (1), go (1)

Table 1: The Lexical Substitution (LexSub) dataset.

- The lexical Substitution (LexSub) dataset (McCarthy and Navigli, 2009).
- 2000 instances of 200 target words in sentential contexts, with paraphrases for each target word generated by up to 6 participants.
- Paraphrases are ranked for their goodness. Can we reproduce scores?

Evaluation of the k-NN disambiguation model

- Given a paraphrase P of a target T in sentence s, compare:
  - the similarity of $act(T, s)$ with P and $act(P, s)$;
  - the similarity of $act(P, s)$ with T and $act(T, s)$.
- Fair results can be obtained with a small value of $k = 10$ (precision around 0.38!)
- Disambiguation in context is a difficult job!