Machine Learning for NLP

Unsupervised Learning

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Unsupervised learning

- In unsupervised learning, we learn without training data.
- The idea is to find a structure in the unlabeled data.
- The following unsupervised learning techniques are fundamental to NLP:
  - dimensionality reduction (e.g. PCA, using SVD or any other technique);
  - clustering;
  - *some* neural network architectures.

Dimensionality reduction

Dimensionality reduction

- Dimensionality reduction refers to a set of techniques used to reduce the number of variables in a model.
- For instance, we have seen that a count-based semantic space can be reduced from thousands of dimensions to a few hundreds:
  - We build a space from word co-occurrence, e.g. *cat - meow: 56* (we have seen *cat* next to *meow* 56 times in our corpus).
  - A complete semantic space for a given corpus would be a $N \times N$ matrix, where N is the size of the vocabulary.
  - N could be well in the hundreds of thousands of dimensions.
  - We typically reduce N to 300-400.

From PCA to SVD

- We have seen that Principal Component Analysis (PCA) is used in the Partial Least Square Regression algorithm for *supervised learning*.
- PCA is *unsupervised* in that it finds ‘the most important’ dimensions in the data just by finding structure in that data.
- A possible way to find the principal components in PCA is to perform Singular Value Decomposition (SVD).
- Understanding SVD gives an insight into the nature of the principal components.

Singular Value Decomposition

- SVD is a **matrix factorisation** method which expresses a matrix in terms of three other matrices:

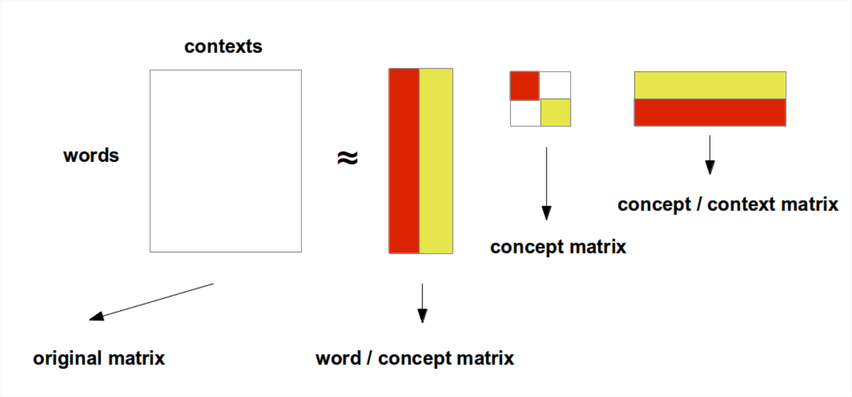
$$A = U\Sigma V^T$$

- U and V are orthogonal: they are matrices such that
  - $UU^T = U^TU = I$
  - $VV^T = V^TV = I$

  I is the identity matrix: a matrix with 1s on the diagonal, 0s everywhere else.

- Σ is a diagonal matrix (only the diagonal entries are non-zero).

Singular Value Decomposition over a semantic space



Taking a linguistic example from distributional semantics, the original word/context matrix A is converted into three matrices U, Σ, V^T, where contexts have been aggregated into ‘concepts’.

The SVD derivation

- From our definition, $A = U\Sigma V^T$, it follows that...
- $A^T = V\Sigma^T U^T$

  See https://en.wikipedia.org/wiki/Transpose for explanation of transposition.
- $A^T A = V\Sigma^T U^T U\Sigma V^T = V\Sigma^2 V^T$
  Recall that $U^T U = I$ because U is orthogonal.
- $A^T A V = V\Sigma^2 V^T V = V\Sigma^2$
  Since $V^T V = I$.
- Note the V on both sides: $A^T A V = V\Sigma^2$
- (By the way, we could similarly prove that $AA^T U = U\Sigma^2$...)

SVD and eigenvectors

- Eigenvectors again! The *eigenvector* of a linear transformation doesn't change its direction when that linear transformation is applied to it:

$$Av = \lambda v$$

A is the linear transformation, and λ is just a scaling factor: v becomes 'bigger' or 'smaller' but doesn't change direction. v is the eigenvector, λ is the eigenvalue.

- Let's consider again the end of our derivation: $A^T AV = V\Sigma^2$.
- This looks very much like a linear transformation applied to its eigenvector (but with matrices)...

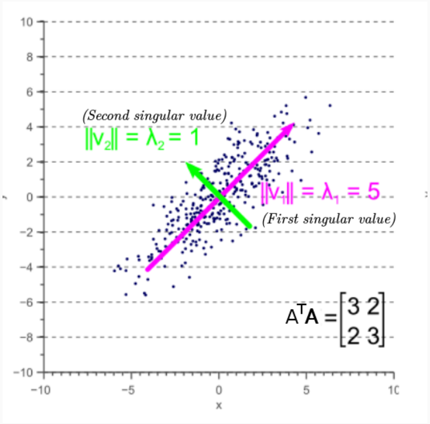
NB: $A^T A$ is a square matrix. This is important, as we would otherwise not be able to obtain our eigenvectors.

SVD and eigenvectors

- The columns of V are the eigenvectors of $A^T A$.
  (Similarly, the columns of U are the eigenvectors of AA^T.)
- $A^T A$ computed over *normalised data* is the *covariance* matrix of A.
  See https://datascienceplus.com/understanding-the-covariance-matrix/.
- In other words, each column in V / U captures variance along one of the (possibly rotated) dimensions of the n-dimensional original data (see last week's slides).

The singular values of SVD

- Σ itself contains the eigenvalues, also known as *singular values*.
- The top k values in Σ correspond to the spread of the variance in the top k dimensions of the (possibly rotated) eigenspace.



http://www.visiondummy.com/2014/04/geometric-interpretation-covariance-matrix/

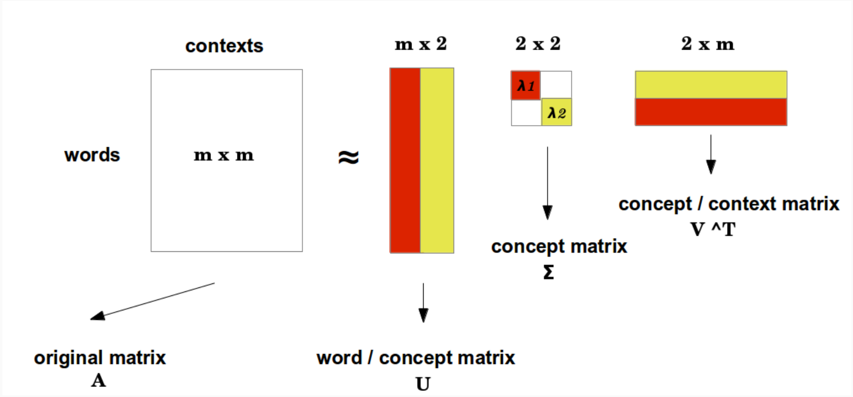
SVD at a glance

- Calculate A^TA = covariance of input matrix A (e.g. word / context matrix).
- Calculate the eigenvalues of A^TA. Take their square roots to obtain the singular values of A^TA (i.e. the matrix Σ).
  If you want to know how to compute eigenvalues, see http://www.visiondummy.com/2014/03/eigenvalues-eigenvectors/.
- Use the eigenvalues to compute the eigenvectors of A^TA. These eigenvectors are the columns of V.
- We had set $A = U\Sigma V^T$. We can re-arrange this equation to obtain $U = AV\Sigma^{-1}$.

Finally... dimensionality reduce!

- Now we know the value of U, Σ and V.
- To obtain a reduced representation of A, choose the top k singular values in Σ and multiply the corresponding columns in U by those values.
- We now have A in a k-dimensional space corresponding to the dimensions of highest covariance in the original data.

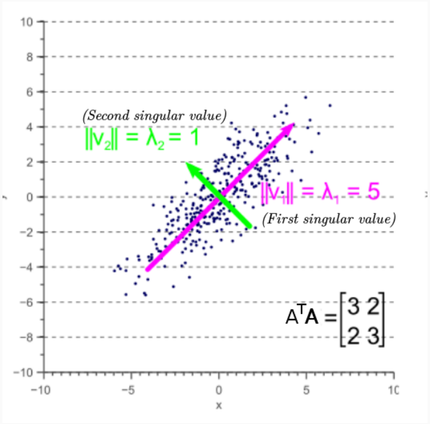
Singular Value Decomposition



What semantic space?

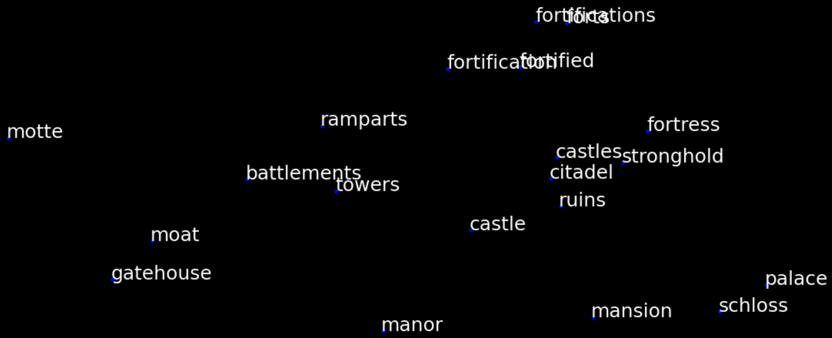
- Singular Value Decomposition (LSA – Landauer and Dumais, 1997). A new dimension might correspond to a generalisation over several of the original dimensions (e.g. the dimensions for *car* and *vehicle* are collapsed into one).
    - + Very efficient (200-500 dimensions). Captures generalisations in the data.
    - - SVD matrices are not straightforwardly interpretable. Can you see why?

The SVD dimensions



Say that in the original data, the x-axis was the context *cat* and the y-axis the context *chase*, what is the purple eigenvector?

PCA for visualisation



Random indexing

Distributional Semantics and Random indexing

Count-based models

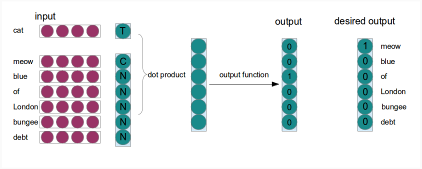
... the walls of the *castle* were built...
... the siege of the *castle* lasted...
... count moved to the *castle* in...

	wall	siege	whale	...
castle	1	1	0	...

Random indexing (RI) is a technique to build distributional semantics space at reduced dimensionality. It works on **count-based** models.

✓

Predictive models



Compare with ‘predictive’ techniques using NNs, which also build a space at reduced dimensionality by virtue of the dimensionality of their input layer.

Why random indexing?

- No distributional semantics method so far satisfies all ideal requirements of a *semantics acquisition model*:
  1. show human-like behaviour on linguistic tasks;
  2. have low dimensionality for efficient storage and manipulation ;
  3. be efficiently acquirable from large data;
  4. be transparent, so that linguistic and computational hypotheses and experimental results can be systematically analysed and explained ;
  5. be **incremental** (i.e. allow the addition of new context elements or target entities). Both standard count models and predictive models fail in that respect.

Random Indexing: the basic idea

- We want to derive a semantic space S by applying a random projection R to a matrix of co-occurrence counts M:

$$M_{p \times n} \times R_{n \times k} = S_{p \times k}$$

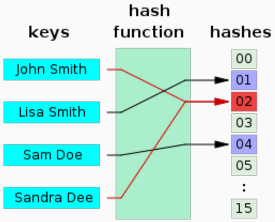
- We assume that $k << n$. So this has in effect dimensionality-reduced the space.
- The above projection can be implemented as an *incremental* process, allowing us to build semantic spaces ‘in real time’.

The components of Random Indexing

- **Random Indexing** uses the principle of **Locality Sensitive Hashing.**
- So we will look in turn at:
  - conventional hashing, and why it does not cater for similarity;
  - the definition of LSH, which solves the similarity problem;
  - how LSH implements the idea of a projection matrix;
  - how a projection can be decomposed into an *incremental* addition operation.

Hashing: definition

- Hashing is the process of converting data of arbitrary size into fixed size signatures (number of bytes).
- The conversion happens through a *hash function*.
- A *collision* happens when two inputs map onto the same *hash (value).*
- Since multiple values can map to a single hash, the slots in the hash table are referred to as *buckets*.



https://en.wikipedia.org/wiki/Hash_function

Hashing strings: an example

- An example function to hash a string s:

$$s[0] * 31^{n-1} + s[1] * 31^{n-2} + ... + s[n-1]$$

  where $s[i]$ is the ASCII code of the ith character of the string and n is the length of s.
- This will return an integer.

Hashing strings: an example

- An example function to hash a string s:

$$s[0] * 31^{n-1} + s[1] * 31^{n-2} + ... + s[n-1]$$

- **A test:** 65 32 84 101 115 116 Hash: 1893050673
- **a test:** 97 32 84 101 115 116 Hash: 2809183505
- **A tess:** 65 32 84 101 115 115 Hash: 1893050672

Modular hashing

- Modular hashing is a very simple hashing function with high risk of collision:

$$h(k) = k \; mod \; m$$

- Let's assume a number of buckets $m = 100$:
  - h(A test) = h(1893050673) = 73
  - h(a test) = h(2809183505) = 5
  - h (a tess) = h(1893050672) = 72

- NB: no notion of similarity between inputs and their hashes. *A test* and *a tess* are very similar but *a test* and *a tess* are not.

Locality Sensitive Hashing

- In ‘conventional’ hashing, similarities between datapoints are not conserved.
- LSH is a way to produces hashes that can be compared with a similarity function.
- The hash function is a projection matrix defining a *random* hyperplane. If the projected datapoint $\vec{v}$ falls on one side of the hyperplane, its hash $h(\vec{v}) = +1$, otherwise $h(\vec{v}) = -1$.

Locality Sensitive Hashing

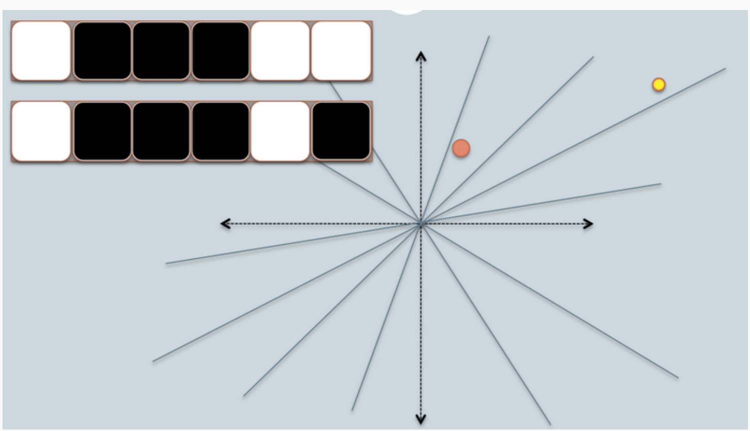
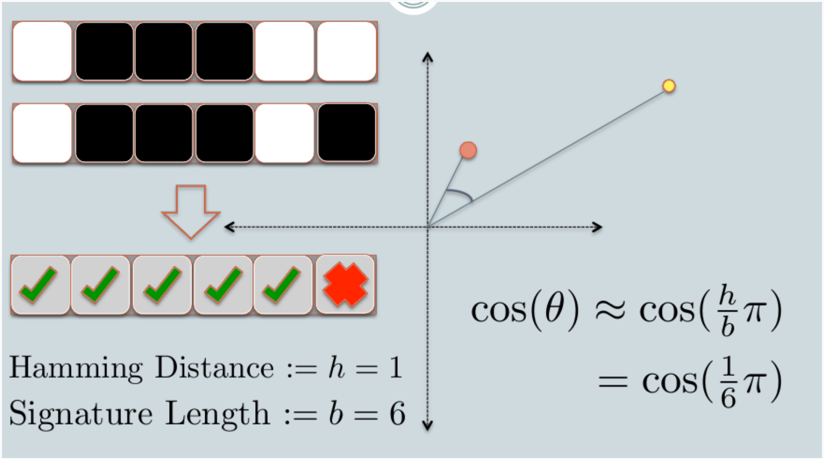


Image from VanDurme & Lall (2010): http://www.cs.jhu.edu/ vandurme/papers/VanDurmeLallACL10-slides.pdf

Locality Sensitive Hashing



$\cos(\theta) \approx \cos(\frac{h}{b}\pi)$
$= \cos(\frac{1}{6}\pi)$
Hamming Distance $:= h = 1$
Signature Length $:= b = 6$

Image from VanDurme & Lall (2010): http://www.cs.jhu.edu/ vandurme/papers/VanDurmeLallACL10-slides.pdf
(The *Hamming distance* between two strings of equal length is the number of positions at which the symbols differ across strings.)

Doing LSH over a semantic space

- The hash value of an input point in LSH is made of all the projections on all chosen hyperplanes:

$$M_{p \times n} \times R_{n \times k} = S_{p \times k}$$

- Say we have k=10 hyperplanes $h_1 ... h_{10}$ (the columns of matrix R) and we are projecting the vector $\overrightarrow{dog}$ with dimensionality n=300 on those hyperplanes:
  - dimension 1 of the new vector is the dot product of $\overrightarrow{dog}$ and h_1: $\sum dog_i h_{1i}$
  - dimension 2 of the new vector is the dot product of $\overrightarrow{dog}$ and h_2: $\sum dog_i h_{2i}$
  - ...
- We end up with a ten-dimensional vector which is the hash of $\overrightarrow{dog}$.

Interpretation of the LSH hash

- Each hyperplane is a discriminatory feature cutting through the data.
- Each point in space is expressed as a function of those hyperplanes.
- We can think of them as new ‘dimensions’ relevant to explaining the structure of the data.
- Random indexing is ‘random’ because the matrix $R_{n \times k}$ is obtained by sampling vectors (hyperplanes) from a Gaussian distribution, in a way that each one is orthogonal to the others.

Random Indexing: incremental LSH

- A random indexing space can be simply and incrementally produced through a two-step process:
  1. Map each *context* item c in the text to a random projection vector (to get R).
  2. Initialise each *target* item t as a null vector. Whenever we encounter c in the vicinity of t we update $\vec{t} = \vec{t} + \vec{c}$.
- The method is extremely efficient, potentially has low dimensionality (we can choose the dimension of the projection vectors), and is fully incremental.

Relating vector addition to random projection

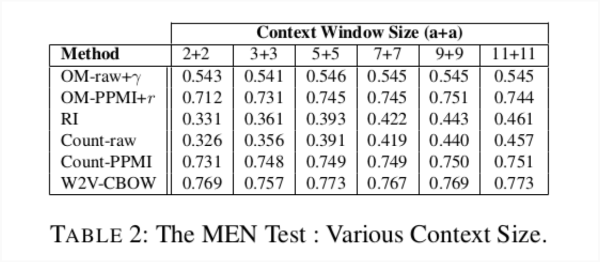
- We said that LSH used a random projection matrix so that $M_{p\times n} \times R_{n\times k} = S_{p\times k}$
- Here's a toy example.

$$\begin{array}{c} \\ t_1 \\ t_2 \end{array} \begin{array}{cc} c_1 & c_2 \\ \multicolumn{2}{c}{\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}} \end{array} \times \begin{array}{c} r \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array} \begin{array}{c} \\ c_1 \\ c_2 \end{array} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

- So we have two random vectors (0) and (1) corresponding to contexts c_1 and c_2.
- We got 3 by computing $2 \times 0 + 3 \times 1$
- This is equivalent to computing $(0 + 0) + (1 + 1 + 1)$
- **So we added the random vectors corresponding to each context for each time it occurred with the target. That's incremental random indexing.**

Is RI human-like?

- Not without adding PPMI weighting at the end of the RI process... (This kills incrementality.)



	Context Window Size (a+a)					
Method	2+2	3+3	5+5	7+7	9+9	11+11
OM-raw+γ	0.543	0.541	0.546	0.545	0.545	0.545
OM-PPMI+r	0.712	0.731	0.745	0.745	0.751	0.744
RI	0.331	0.361	0.393	0.422	0.443	0.461
Count-raw	0.326	0.356	0.391	0.419	0.440	0.457
Count-PPMI	0.731	0.748	0.749	0.749	0.750	0.751
W2V-CBOW	0.769	0.757	0.773	0.767	0.769	0.773

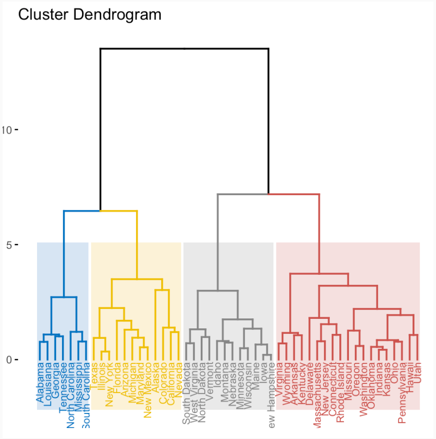
TABLE 2: The MEN Test : Various Context Size.

QasemiZadeh et al (2017)

Clustering

Clustering algorithms

- A clustering algorithm partitions some objects into groups named *clusters.*
- Objects that are similar according to a certain set of features should be in the same cluster.



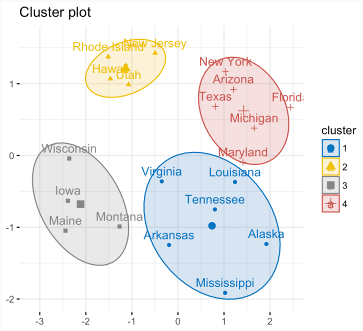
From http://www.sthda.com/english/articles/25-cluster-analysis-in-r-practical-guide/

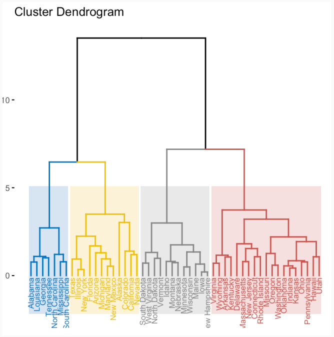
Why clustering

- **Example:**[1] we are translating from French to English, and we know from some training data that:
  - *Dimanche* $\longrightarrow$ *on Sunday*;
  - *Mercredi* $\longrightarrow$ *on Wednesday*;
- What might be the correct preposition to translate *Vendredi* into the English ___ *Friday*? (Given that we haven't seen it in the training data.)
- We can assume that the days of the week form a semantic cluster, which behave in the same way syntactically. Here, clustering helps us **generalise**.

[1] Example from Manning & Schütze, *Foundations of statistical language processing.*

Flat vs hierarchical clustering





Flat clustering

Hierarchical clustering

From http://www.sthda.com/english/articles/25-cluster-analysis-in-r-practical-guide/

Soft vs hard clustering

- In hard clustering, each object is assigned to only one cluster.
- In soft clustering, assignment can be to multiple clusters, or be probabilistic.
- In a *probabilistic* setup, each object has a probability distribution over clusters. $P(c_k|x_i)$ is the probability that object x_i belongs to cluster c_k.
- In a *vector space*, the degree of membership of an object x_i to each cluster can be defined by the similarity of x_i to some representative point in the cluster.

Centroids and medoids

- The *centroid* or *center of gravity* of a cluster c is the average of its N members:

$$\mu_k = \frac{1}{N} \sum_{x \in c_k} x$$

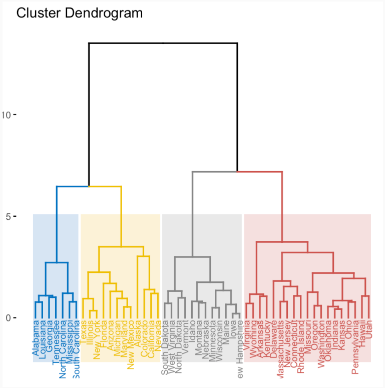
- The *medoid* of a cluster c is a prototypical member of that cluster (its average distance to all other objects in c is minimal):

$$x_{\text{medoid}} = \text{argmin}_{y \in \{x_1, x_2, \cdots, x_n\}} \sum_{i=1}^{n} d(y, x_i)$$

Hierarchical clustering: bottom-up

- Bottom-up agglomerative clustering is a form of hierarchical clustering.
- Let's have n datapoints $x_1 ... x_n$. The algorithm functions as follows:
    - Start with one cluster per datapoint: $c_i := \{x_i\}$.
    - Determine which two clusters are the most similar, and merge them.
    - Repeat until we are left with only one cluster $C = \{x_1 ... x_n\}$.

Hierarchical clustering: bottom-up



NB: The order in which clusters are merged depends on the similarity distribution amongst datapoints.

Hierarchical clustering: top-down

- Top-down divisive clustering is the pendent of agglomerative clustering.
- Let's have *n* datapoints again: $x_1 ... x_n$. The algorithm relies on a *coherence* and a *split* function:
  - Start with a single cluster $C = \{x_1 ... x_n\}$
  - Determine the least coherent cluster and split it into two new clusters.
  - Repeat until we have one cluster per datapoint: $c_i := \{x_i\}$.

Coherence

- **Coherence** is a measure of the pairwise similarity of a set of objects.
- A typical coherence function:

$$Coh(x_{1...n}) = mean\{Sim(x_i, x_j), ij \in 1...n, i < j\}$$

- E.g. coherence may be used to calculate the consistency of topics in topic modelling.

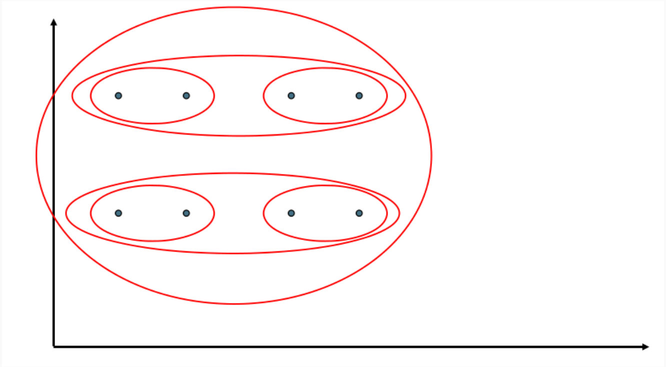
Split

- We need a heuristic to split a cluster into two new clusters.
- Take the point with the *maximum* distance to all other points in the cluster c_n and make it a new cluster c_{n+1}.
- Move all points in c_n which are more similar to c_{n+1}.

Similarity functions for clustering

- **Single link:** similarity of two most similar objects across clusters.
- **Complete link:** similarity of two least similar objects across clusters.
- **Group-average:** average similarity between objects. (Here, the average is over all pairs, within and across clusters.)

Effect of similarity function

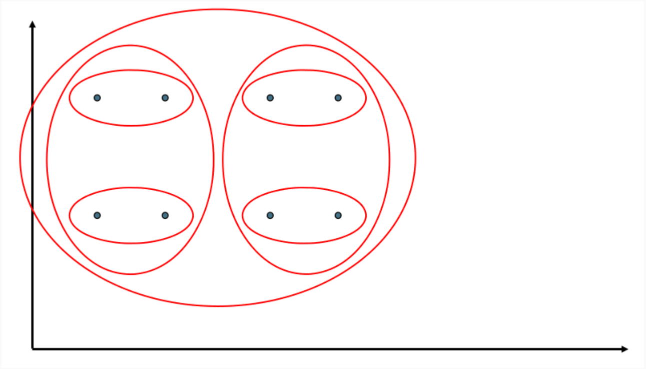


Clustering with single link

Graph taken from Manning, Raghavan & Schütze:

http://www.cs.ucy.ac.cy/courses/EPL660/lectures/lecture12-clustering.pdf

Effect of similarity function



Clustering with complete link

Graph taken from Manning, Raghavan & Schütze:
http://www.cs.ucy.ac.cy/courses/EPL660/lectures/lecture12-clustering.pdf

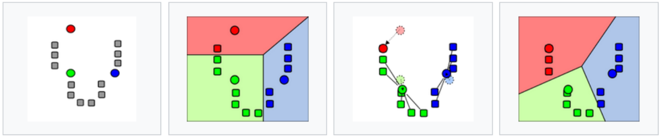
K-means clustering

- K-means is the main flat clustering algorithm.
- Goal in K-means: minimise the *residual sum of squares (RSS)* of objects to their cluster centroids:

$$RSS_k = \sum_{x \in c_k} |x - \mu_k|^2$$

- The intuition behind using RSS is that good clusters should have a) small intra-cluster distances; b) large inter-cluster distances.

K-means algorithm



1. *k* initial "means" (in this case *k*=3) are randomly generated within the data domain (shown in color).

2. *k* clusters are created by associating every observation with the nearest mean. The partitions here represent the Voronoi diagram generated by the means.

3. The centroid of each of the *k* clusters becomes the new mean.

4. Steps 2 and 3 are repeated until convergence has been reached.

Source: https://en.wikipedia.org/wiki/K-means_clustering

Convergence

- Convergence happens when the RSS will not decrease anymore.
- It can be shown that K-means will converge to a local minimum, but not necessarily a global minimum.
- Results will vary depending on seed selection.

Initialisation

- Various heuristics to ensure good clustering:
  - exclude outliers from seed sets;
  - try multiple intialisations and retain the one with lowest RSS;
  - obtain seeds from another method (e.g. first do hierarchical clustering).

Number of clusters

- The number of clusters K is predefined.
- The ideal K will minimise variance within each cluster as well as minimise the number of clusters.
- There are various approaches to finding K. Examples that use techniques we have already learnt:
    - cross-validation;
    - PCA.

Number of clusters

- **Cross-validation:** split the data into random folds and cluster with some *k* on one fold. Repeat on the other folds. If points consistently get assigned to the same clusters (if membership is roughly the same across folds) then *k* is probably right.
- **PCA:** there is no systematic relation between principal components and clusters, but heuristically checking how many components account for most of the data's variance can give a fair idea of the ideal cluster number.

Evaluation of clustering

- Given a clustering task, we want to know whether the algorithm performs well on that task.
- E.g. clustering concepts in a distributional space: *cat* and *giraffe* under ANIMAL, *car* and *motorcycle* under VEHICLE (Almuhareb 2006)
- Evaluation in terms of ‘purity’: if all the concepts in one automatically-produced cluster are from the same category, purity is 100%.

Purity measure

- Given a cluster *k* and a number of classes *i*, purity is defined as

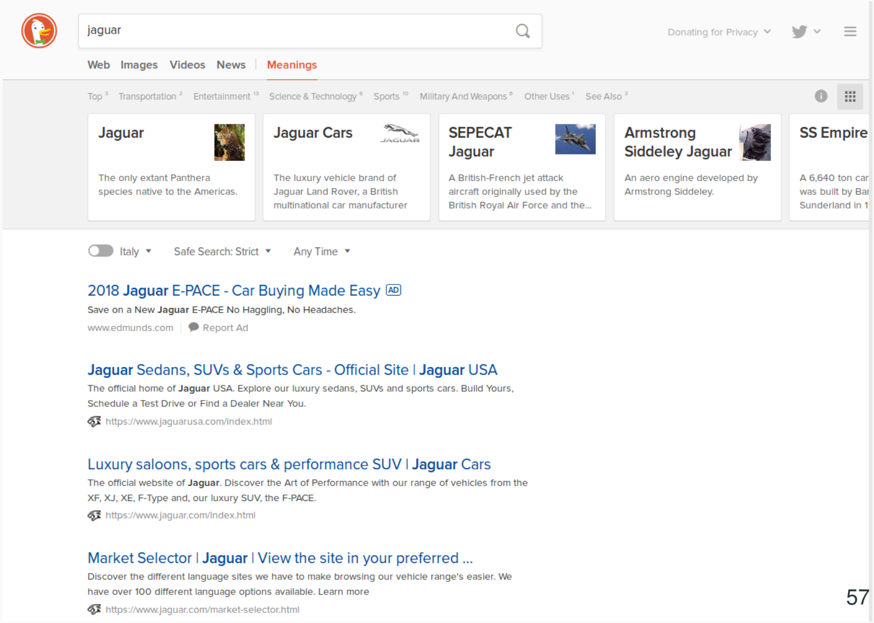
$$P(C_k) = \frac{1}{n_k} max_i(n_k^i)$$

- Example: given the cluster C_1 {A A A T A A A T T A}:

$$P(C_1) = \frac{1}{10} max(7, 3) = 0.7$$

- NB: here, the annotated data is only used for evaluation, *not for training!!* (Compare with k-NN algorithm.)

Clustering in real life



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The only extant Panthera species native to the Americas.

Jaguar Cars
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SEPECAT Jaguar
A British-French jet attack aircraft originally used by the British Royal Air Force and the...

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SS Empire
A 6,640 ton car was built by Bar Sunderland in 1

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